Work: 622 #3

$\frac{1}{a}+\frac{a}{b}+\frac{1}{ab}=1$

$\frac{b}{ab}+\frac{a^2}{ab}+\frac{1}{ab}=1$

$a^2+b+1=ab$

$b>a>1$

$5+b=2b$

$b=5$

$(2,5)$

$10+b=3b$

$b=5$

$(3,5)$

$17+b=4b$

$17=3b$

$26+b=5b$

$26=4b$

$37=5b$

$n^2+2n+2=nb-b$

Answer: $(2,5)$, $(3,5)$

Why: $\frac{1}{a}+\frac{a}{b}+\frac{1}{ab}=1$ can be simplified to $\frac{b}{ab}+\frac{a^2}{ab}+\frac{1}{ab}=1$ and then to $a^2+b+1=ab$. b must be greater than a, or a^2 would alone be more than ab. $a>1$ because $1+b+1=b$ cannot work for any number b. Also, both a and b must be positive integers, as stated in the problem. Therefore, a is at least 2 & b is at least 3. a^2+b+1 can be rewritten as $a^2+1=(a-1)b$. We know that $a^2-1=(a-1)(a+1)$, so then both a^2+1 and a^2-1 divide by $a-1$. The difference between them is 2, so $a-1$ must be a factor of 2, either 1 or 2. If $a-1=1$ or 2, $a=2$ or 3. Substitute those into the equation. $2^2+b+1=2b$, so $4+b+1=2b$, so $5=b$, therefore $(2,5)$ works. $3^2+b+1=3b$, so $9+b+1=3b$, so $10=2b$, so $5=b$, therefore $(3,5)$ works.